Name ____ MTH 1002 - Calculus 2

PRACTICE EDVANCED STANDING EXAM

No books, no notes and no calculators permitted. **Show all work.**

1. Make a sketch and label the intersection points of the region bounded by $x+y=6$ and $y=\dfrac{5}{x}$.
   Set up the integral and evaluate to find the area of the region. [9 pts]

2. Setup the integral to find the volume of the solid generated when the area between the curves of $y=2x$, $y=5x$, $x=0$, and $x=1$ is rotated around the *x-axis*. [9 pts]
(NOTE: You do NOT have to evaluate the integral and get a number. Just setup the integral. That is enough.)

3. Setup the integral to find the volume of the solid generated when the region bounded by the graphs of $y=x^3$, the *x-axis*, $x=4$, and $x=5$ is rotated around the line $x=2$. [9 pts]
(NOTE: You do NOT have to evaluate the integral and get a number. Just setup the integral. That is enough.)

Evaluate the following antiderivatives:

4. $\int xe^{2x}\,dx$ $u =$ $du =$ $v =$ $dv =$

[10 pts]

5. $\int \cos^8 x \sin^3 x\,dx$

[10 pts]

6. $\int \frac{1}{x^2\sqrt{81+x^2}}dx$

[15 points]

If $x =$ then $dx =$

7. $\int \frac{4x^2 + 2x + 3}{x(x^2 + 1)} dx$

[15 pts]

8. $\int_4^{+\infty} \frac{1}{\sqrt{x}} dx$

[9 pts]

Solve the following differential equation by the method of integrating factors.

9. $x\dfrac{dy}{dx}+3y=\dfrac{\sec^2 x}{x^2}$

[12 pts]

10. Setup, but do ***not*** evaluate, an integral to find the arclength of the following:

$x=y^2+5$ from $(6,1)$ to $(14,3)$

[9 pts]

11. Set-up and evaluate (you ***do*** have to complete the integral and get a numeric answer) the definite integral that represents the area of the surface generated by revolving the given curve about the *y*-axis:

$x=3t$ and $y=4t$ with $0\le t\le 2$

[10 pts]

12. Graph the polar functions $r = 3$ and $r = 2 + 2\cos\theta$ and then set-up and evaluate (you ***do*** have to complete the integral and get a numeric answer) an integral to find the area of the region that is outside the circle and inside the cardioid**.**

[16 pts]

13. Find an approximation of the definite integral $\int_0^1 e^{-x^3}\,dx$ by utilizing a series expansion: [12 pts total]

(Hint: Simply write the first four nonzero terms that will be used in the evaluation of the answer. You do ***not*** have to actually convert the resulting fractions to their decimal form or combine them in any way.)

14. Find the first four nonzero terms of the Taylor Series for $f(x) = \cos(x)$ about $x = \frac{\pi}{6}$.

[12 points]

Determine whether the following **geometric series** converges or diverges. If it converges, evaluate the sum.

15. $\sum_{n=0}^{+\infty}\left[2^{3n} \cdot 3^{1-2n}\right]$

[9 points]

Determine whether the following series Converges Absolutely, Converges Conditionally, or Diverges. State the convergence test used (Limit Comparison Test, Ratio Test, etc.) or identify the important features of any series you use (example: "*p*-series with *p*=…") for each step to explain your reasoning.

16. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2+3}{n^3+1}$

[9 points]

Determine the interval of convergence of the following Power Series:

17. $$\sum_{n=1}^{+\infty} \frac{1}{(n+3)\cdot 2^n}(x-1)^n$$

[13 points]

18. Initially, a tank contains 200 gal of water with 10 lbs of salt dissolved in it.
A solution containing 4 lbs of salt/gal is then poured into the tank at a rate of 3 gal/min and the mixed solution is drained from the tank at the same rate.

(a) Find an initial value problem whose solution is $y(t)$, the amount of salt (in lbs) at time t. (3 pts)

(b) Solve he differential equation for $y(t)$ (4pts)

(c) Determine the amount of salt in the tank after 60 min. (3 pts)

[Hint: Mixing Model: $\frac{dy}{dt} = \textit{rate in} - \textit{rate out}$]

Direct Comparison Test:

If $a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

If $b_n \le a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.

Limit Comparison Test: If $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ and $L > 0$ and finite, then both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ behave the same. That is, both series converge or both diverge.

Divergence Test: In order for a series $\sum_{n=0}^{\infty} a_n$ to converge, the terms a_n must be going to zero. Just because the terms in the series are going to zero does not automatically mean that it converges. However, if they go to any other limit besides zero, the series definitely *diverges*:

If $\sum_{n=0}^{\infty} a_n$ and $\lim_{n\to\infty} a_n = L \ne 0$, then $\sum_{n=0}^{\infty} a_n$ diverges.

Integral Test: Let $\sum_{n=1}^{+\infty} u_n$ be a series with positive terms, and let *f(x)* be the function that results when *n* is replaced by *x*. If f is decreasing and continuous on the interval $[1, +\infty)$, then either $\sum_{n=1}^{+\infty} u_n$ and $\int_1^{+\infty} f(x)dx$ both converge or both diverge.

Alternating Series Test for either form $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ or $\sum_{n=1}^{\infty} (-1)^n a_n$

Check that the following two conditions are met:

(1) Ignoring the $+/-$ signs, check that the terms are decreasing. That is, make certain that $a_n \ge a_{n+1}$

(2) Check that the terms are heading to zero. That is, make sure that $\lim_{n\to\infty} a_n = 0$

If *both* conditions are satisfied, then the alternating series converges.

Ratio Test for Absolute Convergence:

If $\lim_{n\to\infty} \frac{|u_{n+1}|}{|u_n|} = L$ and $L < 1$, then the series $\sum_{n=1}^{\infty} u_n$ converges absolutely. If $L > 1$, then the series diverges.
If $L = 1$, then the test provides no useful information and another test must be used.

Root Test for Absolute Convergence:

If $\lim_{n\to\infty} \sqrt[n]{|u_n|} = L$ and $L < 1$, then the series $\sum_{n=1}^{\infty} u_n$ converges absolutely. If $L > 1$, then the series diverges.
If $L = 1$, then the test provides no useful information and another test must be used.

A series $\sum_{n=1}^{\infty} u_n$ *converges absolutely* if $\sum_{n=1}^{\infty} |u_n|$ converges.

A series $\sum_{n=1}^{\infty} u_n$ *converges conditionally* if $\sum_{n=1}^{\infty} |u_n|$ diverges and $\sum_{n=1}^{\infty} u_n$ converges.