1. A bead of mass $m$ slides on a frictionless wire that is bent into the shape of a parabola, $z = \beta q^2$, as shown. The wire is in a gravitational field $g$ with the $z$ axis oriented vertically. The wire is also rotating about the $z$ axis at fixed angular velocity $\Omega$.

(a) Using $q$ as the generalized coordinate, calculate the Lagrangian.

(b) Determine the equation of motion for the mass.

(c) Is there a steady-state solution for which $\dot{q} = \ddot{q} = 0$, other than $q = 0$?

2. Recall the definition of the Hamiltonian

$$H = \sum_\alpha p_\alpha \dot{q}_\alpha - L,$$

with $p_\alpha = \partial L / \partial \dot{q}_\alpha$ and $L = L(q, \dot{q}, t)$. Prove that $H$ is a constant of the motion under the appropriate conditions, being sure to identify clearly what those conditions are.

Now consider a particle in a central field $U(r)$.

(a) Use plane polar coordinates to find the Hamiltonian equations of motion for the particle. [Hint: recall that $H = H(q, p)$]

(b) Is there a cyclic coordinate? If so, what is the associated conserved quantity?

(c) Write the radial equation of motion.

3. A bicycle travels with constant speed around a circular track of radius $r$ without leaning. What is the acceleration of the highest point on one of its wheels relative to the ground?
4. A mass \( m_1 \), with initial velocity \( v_0 \), strikes a mass-spring system \( m_2 \), initially at rest but able to recoil. The spring is massless with spring constant \( k \). There is no friction. (The mass \( m_1 \) strikes the spring attached to \( m_2 \).)

(a) What is the maximum compression of the spring?

(b) If, long after the collision, both objects travel in the same direction, which mass is the greatest? What are the final velocities \( v_1 \) and \( v_2 \) of \( m_1 \) and \( m_2 \), respectively?

5. Consider a particle of mass \( m \), charge \( e \), in a uniform electric field \( E \). The particle’s initial velocity is in the +x direction and the electric field points in the +y direction.

(a) Use Newton’s second law to write down the equations motion; integrate the equations of motion to obtain the momenta in the plane of motion

(b) Write down the total energy of the particle, which may be moving relativistically, ignoring the potential energy due to the field

(c) The work done by the electric field changes the energy of the particle at a rate of \( eE \cdot v \). Calculate the amount of energy generated by the electric field.

6. A skier weighing 90 kg starts from rest down a hill inclined at 17°. He skis 100 m down the hill and then coasts for 70 m along level snow until he stops.

(a) Find the coefficient of kinetic friction between the skis and the snow.

(b) What velocity does the skier have at the bottom of the hill?