Electricity & Magnetism Qualifier Exam Spring 2011

**DISCLAIMER:** This sample PhD qualifier exam only demonstrates the typical level of question that may be posed by the GRASP. Students must not infer anything regarding the content of their exam based on these examples; questions may be drawn from the full range of the topic.

1. A current $I$ flows down a long straight wire of radius $a$. If the wire is made of linear material with susceptibility $\chi_m$, and the current is distributed uniformly, what is the magnetic field inside the wire? Find all the bound currents. What is the net bound current flowing down the wire?

2. A plane-polarized electromagnetic wave is incident on a perfect conductor at an angle $\theta_I$. The electric field is given by

\[ E_I = E_0 I \Re\{\exp[i(k_I \cdot r - \omega t)]\}. \]

$E$ is in the plane of incidence as shown below. Starting with the boundary conditions imposed on an electromagnetic field by a conductor, derive the following properties of the reflected wave: direction of propagation, amplitude, and phase.

3. The energy per unit volume outside of a uniform charge is given by $0.5\varepsilon_0 E^2$, where $E$ is the electric field strength. Derive an expression for the total electrostatic energy outside an electron and, using rest mass energy, determine the radius of the electron.
4. Consider two donut-shaped permanent magnets with magnetization parallel to the vertical axis \( z \) (the vertical direction along the gravitational acceleration vector \( g \)), which slide without friction on a vertical rod. Treat the magnets as dipoles, with mass \( M \) and dipole moment \( m \). If you put the magnets back-to-back (NS-SN), the upper magnet will float – the magnetic force upward balancing the gravitational force downward. At what height \( z \) does it float? [Extra Credit] Determine the angular frequency of small oscillations about the equilibrium height. *Hint: use the Taylor expansion for small oscillations around the equilibrium point.*

5. An electric dipole has a charge \( q \) at \( z = a/2 \) and charge \( -q \) at \( z = -a/2 \). The potential at \( r \) (when \( r \gg a \)) is given by \( V = (kqa \cos \theta)/r^2 \) where \( \theta \) is the angle between the z-axis and \( r \). Using spherical polar coordinates, show that the ratio of the electric field components \( E_\theta/E_r = 0.5 \tan \theta \).