1. (a) Write the general definition of the derivative for a function \( f(x) \)

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

[4 pt setup]

(b) Find \( f'(x) \) by using the definition of the derivative with the following function:

\( f(x) = \frac{1}{x} \)

\[
f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}
\]

[3 pt setup]

\[
= \lim_{h \to 0} \frac{x - (x+h)}{x(x+h)h} = \lim_{h \to 0} \frac{-h}{x(x+h)h}
\]

[3 pt expand & cancel]

\[
= \lim_{h \to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2}
\]

[2 pts factor & simplify]

[2 pts take lim & answer]

2. Find the derivative: \( f(x) = x^3 \tan(2x - 1) \)

\[
f'(x) = 3x^2 \cdot \tan(2x - 1) + x^3 \cdot \sec^2(2x - 1) \cdot 2
\]

[3 pts prod rule & 3 pts poly deriv & 3 pts trig derive & 3 pts chain rule]

3. Find the derivative: \( f(x) = e^{x^3} + \ln(\sec x) + \csc(\ln x) \)

\[
f'(x) = 3x^2 e^{x^3} + \frac{\sec x \tan x}{\sec x} - \csc(\ln x) \cot(\ln x) \cdot \frac{1}{x} = 3x^2 e^{x^3} + \tan x - \frac{\csc(\ln x) \cot(\ln x)}{x}
\]

(don’t need to simplify any terms)
4. Find the derivative $\frac{dy}{dx}$ for the following: $x^2 + y^3 = ye^{5x}$

\[2x + 3y^2 \frac{dy}{dx} = \frac{dy}{dx} \cdot e^{5x} + y \cdot 5e^{5x}\]
\[3y^2 \frac{dy}{dx} - e^{5x} \frac{dy}{dx} = 5ye^{5x} - 2x\]
\[(3y^2 - e^{5x}) \frac{dy}{dx} = 5ye^{5x} - 2x\]
\[\frac{dy}{dx} = \frac{5ye^{5x} - 2x}{3y^2 - e^{5x}}\]

[6 pts derive & 2 pts rearrange & 2 pts ans]

5. Find the derivative: $f(x) = \frac{\sin x}{x} + \sin^{-1} x + \sinh x$

\[f'(x) = \frac{x \cos x - \sin x}{x^2} + \frac{1}{\sqrt{1-x^2}} + \cosh x\]

[9 pts]

3 pts quot rule & 3 pts inv trig & 3 pts hyperbolic trig

6. Find the derivative: $y = x^{3x}$

\[\frac{dy}{dx} = x^{3x} \ln x + \frac{3x}{x^{3x}}\]
\[\ln y = \ln(x^{3x})\]
\[\frac{1}{y} \cdot \frac{dy}{dx} = 3 \ln x + 3x \cdot \frac{1}{x}\]
\[\frac{dy}{dx} = y(3 \ln x + 3)\]
\[\frac{dy}{dx} = x^{3x}(3 \ln x + 3)\]

[12 pts]
7. A toy car moves along a straight track during time $0 \leq t \leq 4$. It’s position at any time from a fixed point along the track is given by $s(t) = t^3 - 3t^2$
Answer the following about the motion of the car.
(Note: The time $t$ is measured in minutes and distance $s$ in inches.)

(a) What is the position, velocity, and acceleration of the car at the time $t = 3$ minutes?

$$s(t) = t^3 - 3t^2 \Rightarrow s(3) = 27 - 27 = 0 \quad [2pts]$$

$$v(t) = 3t^2 - 6t \Rightarrow v(3) = 27 - 18 = +9 \quad [2pts]$$

$$a(t) = 6t - 6 \Rightarrow a(3) = 18 - 6 = +12 \quad [2pts]$$

(b) At what time does the car come to a stop?

$$v(t) = 0 \Rightarrow 3t^2 - 6t = 3(t-2) = 0 \text{ at } t = 0 \text{ and } t = 2 \quad [4pts]$$

8. A 5 ft ladder is leaning against a wall and starts to slide. How fast is the bottom edge of the ladder moving along the floor when the top corner of the ladder is 3 ft up the wall and sliding down the wall at a rate of 8 ft/sec?

$$x^2 + y^2 = 5^2 \quad [2 pts]$$

$$x^2 + y^2 = 25 \Rightarrow x^2 + 9 = 25 \Rightarrow x^2 = 16 \Rightarrow x = 4$$

$$\begin{align*}
x &= 4 & \frac{dx}{dt} &= ? \\
y &= 3 & \frac{dy}{dt} &= -8
\end{align*}$$

$$\begin{align*}
x^2 + y^2 &= 25 \\
2x \frac{dx}{dt} + 2y \frac{dy}{dt} &= 0 \\
x \frac{dx}{dt} + y \frac{dy}{dt} &= 0 \quad [5 pts]
\end{align*}$$

$$\begin{align*}
\frac{4}{\frac{dx}{dt}} &= 24 \\
\frac{dx}{dt} &= +6 \frac{ft}{sec} \quad [5 pts]
\end{align*}$$

9. Use L’Hôpital’s Rule to evaluate the following limit:

$$\lim_{x \to 0} \frac{x^3 + 5\sin x}{x \cos x} = \lim_{x \to 0} \frac{3x^2 + 5\cos x}{\cos x - x \sin x} = \frac{0 + 5}{1 - 0} = \frac{5}{1} = 5 \quad [6 pts \text{ L’Hôp} \ [3 \text{ pts num} & 3 \text{ pts denom}] & 2 \text{ pts ans}$$
10. Graph the following Rational Function:

\[ f(x) = \frac{36(x-1)}{x^2} \]

[16 pts]

Hint: \( f'(x) = \frac{36(2-x)}{x^3} \) and \( f''(x) = \frac{72(x-3)}{x^4} \)

(Use calculus to find the locations of any important points [maxs, mins, pts of inflection] and label them on the graph.)

\[ f'(x) = \frac{36(2-x)}{x^3} = \frac{AB}{C} \]

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[2 pt first deriv & chart]

\[ f''(x) = \frac{72(x-3)}{x^4} = \frac{AB}{C} \]

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[2 pt second deriv & chart]

\[ @ x = 2 \Rightarrow \frac{36(2-1)}{2^2} = \frac{36}{4} = 9 \Rightarrow (2, 9) \]

\[ @ x = 3 \Rightarrow \frac{36(3-1)}{3^2} = \frac{36}{3} = 4 \cdot 2 = 8 \Rightarrow (3, 8) \]

[3 pts assemble pieces]

[1 pt get pts]

[4 pts H.A. & V.A.]
11. A box with a closed top is going to be manufactured so that its base is a square and its volume will be 100 cm$^3$. If the material to make the top and bottom of the box cost $50 per square cm and the material for the sides costs $4 per square cm, find the dimensions that will minimize the cost of the box.

\[ x^2 y = 100 \quad \text{[2 pts constraint eq]} \]

\[ y = \frac{100}{x^2} \quad \text{[1 pt rewrite]} \]

\[ C = 50\left(2x^2\right) + 4\left(4xy\right) = 100x^2 + 16xy \quad \text{[2 pts cost]} \]

\[ C = 100x^2 + 16x\left(\frac{100}{x^2}\right) = 100x^2 + 1600x^{-1} \quad \text{[2 pts rewrite]} \]

\[ C = 200x - 1600x^{-2} = \frac{200x^3 - 1600}{x^2} \quad \text{[2 pts derive]} \]

\[ 200x^3 - 1600 = 0 \Rightarrow x^3 = \frac{1600}{200} = \frac{16}{2} = 8 \Rightarrow x = \sqrt[3]{8} = 2 \]

C.P.: $x = 2$ and $x = 0 \Rightarrow$ discard $x = 0$

\[ x = 2 \quad \text{[2 pts x-value]} \]

\[ y = \frac{100}{4} = 25 \quad \text{[1 pt y-value]} \]

12. Find the exact area under the curve $f(x) = 2x + 1$ over the interval $[a, b]$, where $x_i$ is the right endpoint of each equal subinterval, given $a = 1$ and $b = 3$.

[16pts]

Hint – Evaluate the limit: $\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x$

\[ \sum_{i=1}^{n} (1) = n \quad \sum_{i=1}^{n} (i) = \frac{n(n+1)}{2} \quad \sum_{i=1}^{n} (i^2) = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^{n} (i^3) = \left[\frac{n(n+1)^2}{2}\right]^2 \]

\[ \Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n} \quad \text{[2 pts]} \]

\[ x_i = 1 + i \cdot \frac{2}{n} = 1 + \frac{2}{n}i \]

\[ y = f(x_i) = 2\left(1 + \frac{2}{n}i\right) + 1 = 2 + \frac{2}{n}i + 1 = 3 + \frac{2}{n}i \quad \text{[4 pts]} \]

\[
\text{Area} = \lim_{n \to \infty} \left( \sum_{i=1}^{n} f(x_i) \Delta x \right) = \lim_{n \to \infty} \left( \sum_{i=1}^{n} \left(3 + \frac{2}{n}i\right) \frac{2}{n} \right) = \lim_{n \to \infty} \left( \sum_{i=1}^{n} \left[\frac{6}{n} + \frac{8}{n^2}i\right] \right) \\
= \lim_{n \to \infty} \left( \sum_{i=1}^{n} \left[\frac{6}{n}\right] + \sum_{i=1}^{n} \left[\frac{8}{n^2}i\right] \right) = \lim_{n \to \infty} \left( \frac{6}{n} \cdot \frac{n}{1} + \frac{8}{n^2} \cdot \frac{n(n+1)}{2} \right) \\
= \lim_{n \to \infty} \left(6 + \frac{4(n+1)}{n}\right) = \lim_{n \to \infty} \left(6 + \frac{4n+4}{n}\right) = 6 + \frac{4}{1} = 6 + 4 = 10
\]

[2 pts setup lim & 4 pts breakup sigmas & 4 pts final ans]
13. Evaluate the indefinite integral: \( \int (50x^4 + 10x^3 + 12\sqrt{x}) \, dx \)

\[ \int (50x^4 + 10x^3 + 12x^{\frac{1}{2}}) \, dx = 10x^5 + \frac{5}{2}x^4 + 8x^{\frac{3}{2}} + C \]

[1 pt rewrite root & 2 pts \( x^\frac{1}{2} \)-term & 4 pts \( x^4 \)-term & 4 pts \( x^{3/2} \)-term & 1 pt “C”]

Setup a definite integral and find the area of the indicated regions:

14. 

\[ \int_1^2 x^2 \, dx = \left( \frac{1}{3}x^3 \right)_1^2 = \left( \frac{2^3}{3} \right) - \left( \frac{1}{3} \right) = \frac{8}{3} - \frac{1}{3} = \frac{7}{3} \]

2 pts setup & 2 pts antiderive & 1 pt plugin/answer

15. 

\[ \int_0^\pi \sin x \, dx = (-\cos x)_0^\pi = (-\cos(\pi)) - (-\cos(0)) = (-1) - (-1) = 1 + 1 = 2 \]

1 pt setup & 2 pts antiderive & 2 pts plugin/answer

16. Evaluate the following: \( \frac{d}{dx} \left( \int_4^{x^3} e^{y^2} \, dy \right) \)

\[ e^{(x^3)^2} \cdot 3x^2 - e^{(4)^2} \cdot 0 = e^{x^6} \cdot 3x^2 - 0 = 3x^2 e^{x^6} \]

6 pts cancel and try to plugin lim & 4 pts remember to plugin to \( dt \)
17. Evaluate the indefinite integral: \[ \int [24 \sin^2 (4x) \cos (4x)] \, dx \]

[14 pts] 2 pts \( u \)-choice
\( u = \sin (4x) \) \( \Rightarrow \) \( du = 4 \cos (4x) \, dx \) \( \Rightarrow \) \( dx = \frac{du}{4 \cos (4x)} \)

6 pts rewrite/simplify 4 pts \( u \)-antideriv 2 pts back to \( x \)'s
\[ \int 24 (\sin (4x))^2 \cos (4x) \, dx = \int 24 u^2 \cos (4x) \, \frac{du}{4 \cos (4x)} = \int 6 u^2 \, du = 2 u^3 + C = 2 (\sin (4x))^3 + C \]

18. Evaluate the definite integral: \[ \int_0^1 [8x(x^2 + 1)] \, dx \]

[14 pts] 1 pt \( u \)-choice
\( u = x^2 + 1 \) \( \Rightarrow \) \( du = 2x \, dx \) \( \Rightarrow \) \( dx = \frac{du}{2x} \)
\[ x = 1 \Rightarrow u = 1^2 + 1 = 2 \]
\[ x = 0 \Rightarrow u = 0^2 + 1 = 1 \]

5 pts rewrite/simplify 3 pts \( u \)-antideriv 3 pts change lim or back to \( x \)'s 2 pts ans
\[ \int_{x=0}^{x=1} [8x(x^2 + 1)^3] \, dx = \int_{u=1}^{u=2} [8xu^3] \, \frac{du}{2x} = \int_{u=1}^{u=2} [4u^3] \, du = u^4 \Bigg|_{u=1}^{u=2} = (2^4) - (1^4) = 16 - 1 = 15 \]